

Out of Equilibrium Dynamics of Supersymmetry at High Energy Density

J. Baacke^(a), D. Cormier^(b), H. J. de Vega^(c), and K. Heitmann^(d)

(a) Institute for Physics, University of Dortmund, D-44221 Dortmund, Germany

(b) Centre for Theoretical Physics, Sussex University, Falmer, Brighton, BN1 9QH, England

(c) Laboratoire de Physique Théorique et Hautes Energies, Université Pierre et Marie Curie (Paris VI) et Denis Diderot (Paris VII), Tour 16, 1er. étage, 4, Place Jussieu, 75252 Paris cedex 05, France

(d) T-8, Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

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Abstract

We investigate the out of equilibrium dynamics of global chiral supersymmetry at finite energy density. We concentrate on two specific models. The first is the massive Wess-Zumino model which we study in a selfconsistent one-loop approximation. We find that for energy densities above a certain threshold, the fields are driven dynamically to a point in field space at which the fermionic component of the superfield is massless. The state, however is found to be unstable, indicating a breakdown of the one-loop approximation. To investigate further, we consider an $O(N)$ massive chiral model which is solved exactly in the large N limit. For sufficiently high energy densities, we find that for late times the fields reach a nonperturbative minimum of the effective potential degenerate with the perturbative minimum. This minimum is a true attractor for $O(N)$ invariant states at high energy densities, and this provides a mechanism for determining which of the otherwise degenerate vacua is chosen by the dynamics. The final state for large energy density is a cloud of massless particles (both bosons and fermions) around this new non-perturbative supersymmetric minimum. By introducing boson masses which softly break the supersymmetry, we demonstrate a see-saw mechanism for generating small fermion masses. We discuss some of the cosmological implications of our results.

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I. INTRODUCTION

Supersymmetry [1], by nature of its Grassman transformation parameter, behaves differently from ordinary symmetries under the effects of finite temperature, as was first recognized over 20 years ago by Das and Kaku [2] and has been studied since by a number of authors [3–13]. In particular, it was shown that unbroken supersymmetry in equilibrium at zero temperature becomes broken at finite temperatures [7,8]. As usual, such breaking of the continuous symmetry is necessarily signaled by the appearance of a massless particle, the Goldstone. Only in this case the Goldstone is a fermion rather than a boson, due to the fact that the symmetry transformation parameter is a Grassman variable; the residual flat direction in field space must correspond to a fermionic degree of freedom. The Goldstone fermion is referred to as the Goldstino.

The fundamental reason why supersymmetry becomes broken at finite temperature is easy to understand. At finite temperature, fermions and bosons obey different statistics. This means that while the Lagrangian may admit a supersymmetry between boson and fermion components, the way in which these components are populated at finite temperature breaks any such symmetry. In equilibrium, the only state which transforms supersymmetrically is the zero temperature supersymmetric ground state.

These issues are relevant for a number of reasons. We see no supersymmetric partners to the fermions nor to the bosons in nature. Hence, if we are to assume that supersymmetry is fundamental, then it must be broken. Supersymmetry breaking is an appealing possibility which raises the question of the nature and identification of the resulting Goldstino.

The possible breaking of supersymmetry is also relevant to the early universe. In particular, many popular inflation models are based on supersymmetry in some form [14]. However, inflation occurs very far from thermal equilibrium and while supersymmetric models are understood at zero and finite temperature, the non-equilibrium dynamics of such models have yet to be properly studied. And even when supersymmetric models are considered in the context of inflation, it is common practice to discard some of the degrees of freedom as irrelevant. However, if the results in thermal equilibrium are to be a guide, this may be a dangerous thing to do. Processes such as the breaking of supersymmetry and the consequent appearance of massless degrees of freedom might also occur out of equilibrium as the available energy is distributed differently among fermions and bosons. This may be particularly relevant for the process of reheating after inflation, for which light fermions may play an important role [15–19].

We examine these issues by means of explicit numerical solutions of supersymmetric toy models allowed to evolve far from equilibrium. First results describing the dynamics of $O(N)$ chiral supersymmetry at finite energy were published recently [20]. Although the primary applications of interest are cosmological, we simplify the analysis by restricting ourselves to Minkowski spacetime.

We begin our analysis with a study of the massive Wess-Zumino model in a self-consistent one-loop approximation. We find two distinct dynamical regimes. At sufficiently low energy density, the system remains near one of the two supersymmetric vacua of the model. The non-zero energy is taken up in the oscillation of the field zero modes about their perturbative vacuum values. Some particles are produced, but there is no obvious massless fermion in the spectrum over the lifetime of the numerical simulations. These states are continuously

connected to the zero temperature vacuum states in the sense that as one reduces the initial energy density of the system toward zero, the system approaches the zero temperature vacuum without any changes in the qualitative behavior of the system.

The second regime is quite different. When the system exceeds a certain critical energy density such that the fields can sufficiently sample the region separating the two perturbative minima, the system becomes driven to the point halfway between the two minima, i.e., to the local maximum of the effective potential. What is special about this point is that the mass of the fermion field precisely vanishes.

There are a couple things we learn from this behavior. First, this provides another instance of the dangers of relying solely on equilibrium constructs such as the effective potential when dealing with systems far from equilibrium [21,22]. The perturbative effective potential alone would never cause one to expect that the system would approach such a point. Second, with sufficient energy to reach the relevant point in field space, the system will invariably approach a state for which the fermion field is massless. Numerical tests of the system indicate that this behavior is obtained for arbitrarily high energy densities.

However, we also find that this state has an instability to this order in perturbation theory as it includes a scalar field with negative mass squared, corresponding to a so-called spinodal instability. This is expected from being at the local maximum of the effective potential. This is a signal of the breakdown of perturbation theory and an indication that the state which eventually forms is non-perturbative in nature. This unfortunately limits our study in the one component model to the early time behavior before the instability becomes important.

In order to proceed beyond the perturbative Wess-Zumino model, we introduce a model with an internal $O(N)$ symmetry, which can be solved exactly, even into the non-perturbative regime, in the limit $N \rightarrow \infty$. While an $O(\infty)$ field theory may not be realistic, it provides a useful self-consistent toy model for the study of possible characteristics of realistic theories – particularly those with continuous symmetries – and may also be considered as a first approximation to systems with moderate values of N . Such techniques have a long history in field theory [23] in general as well as in supersymmetric theories [24]. In the case of purely scalar models, large N studies provide concrete examples of the non-perturbative symmetry breaking processes leading to Goldstone bosons [22,25] and to a dynamical formation of a flat potential related to the Maxwell construction of equilibrium thermodynamics [26].

Our results again indicate two regimes for states invariant under the $O(N)$ symmetry. As with the one-loop Wess-Zumino model, there is a continuous spectrum of states at sufficiently low energy densities which are connected to the perturbative $O(N)$ symmetric vacua. For sufficiently high energy densities, the system again is driven to a point in field space for which the fermion modes are massless. This point, $\phi = -m/\lambda$ which acts as an attractor to the dynamics is a nonperturbative minimum of the effective potential degenerate with the perturbative minimum. The final state obtained by real time evolution for large energy density is a cloud of massless particles (both bosons and fermions) around this new nonperturbative minimum.

There are several interesting facts about this system that deserve mention. First, we see that the dynamics effectively chooses the vacuum for the system during the high energy density phase. In a cosmological context, once such a vacuum is chosen, the universe would stay in that vacuum indefinitely. We therefore see a mechanism for choosing one vacuum

over another in spite of the fact that they are degenerate in energy. Second, we find that all particles are massless for arbitrarily high energy density. This is to be contrasted with the general, non-supersymmetric case for which continuous symmetries are always restored at high enough energy density and particles become massive. The study of the spontaneously broken $O(N)$ scalar theory provides a good example of this more usual behavior [26]. In the present case, however, the supersymmetry acts to protect the kind of Goldstone phase we find with all particles, bosons and fermions, being massless.

As a final example, we examine what occurs when the supersymmetry of the $O(N)$ model is softly broken by the introduction of additional small scalar mass terms. The result is that while there is still a set of massless bosons in the high energy density phase, their superpartner bosons and fermions gain masses. While the bosons gain a mass proportional to the soft breaking mass scale, there is a see-saw mechanism for the fermions which gives them a mass proportional to the square of the soft breaking mass scale divided by the overall scale of supersymmetry, providing a very natural mechanism for producing very light fermions which could be relevant to neutrino mass generation.

We continue in the next section with the introduction of the Wess-Zumino model and provide the renormalized one-loop equations of motion appropriate to an out of equilibrium study. This is followed by our numerical results for the model, showing the two distinct regimes at low and high energies. In Section III, we introduce the $O(N)$ model and again provide our numerical results. In Section IV, we include soft supersymmetry breaking into the $O(N)$ Lagrangian and show how the various fields gain non-zero masses. Our conclusions are provided in Section V. We also present two appendices with details of the renormalization procedures used in the non-equilibrium formalism.

II. THE WESS-ZUMINO MODEL TO ONE LOOP ORDER

A. model and equations of motion

We consider the supersymmetric Wess-Zumino model [27]. It is based on a single chiral super-multiplet S with superpotential [1]

$$W(S) = \frac{1}{2}mS \cdot S + \frac{1}{3}\lambda S \cdot S \cdot S, \quad (2.1)$$

This can be broken down into component fields via $S = (\mathcal{A}, \mathcal{B}; \psi; F, G)$, where \mathcal{A} is a scalar, \mathcal{B} is a pseudo-scalar, ψ is a Majorana fermion, and F and G are scalar and pseudo-scalar auxiliary fields respectively. After eliminating the auxiliary fields, the Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}\partial_\mu \mathcal{A} \partial^\mu \mathcal{A} + \frac{1}{2}\partial_\mu \mathcal{B} \partial^\mu \mathcal{B} - \frac{1}{2}m^2 (\mathcal{A}^2 + \mathcal{B}^2) - \frac{1}{2}\lambda m \mathcal{A} (\mathcal{A}^2 + \mathcal{B}^2) - \frac{1}{8}\lambda^2 (\mathcal{A}^2 + \mathcal{B}^2)^2 \\ & + \frac{i}{2}\bar{\psi} \not{\partial} \psi - \frac{1}{2}m\bar{\psi}\psi - \frac{\lambda}{2}\mathcal{A}\bar{\psi}\psi - \frac{i\lambda}{2}\mathcal{B}\bar{\psi}\gamma_5\psi. \end{aligned} \quad (2.2)$$

In order to follow the dynamics, it is convenient to break up the scalar field \mathcal{A} into its expectation value and small fluctuations about that value:

$$\mathcal{A} = \phi + A, \quad \phi = \langle \mathcal{A} \rangle, \quad \langle A \rangle = 0. \quad (2.3)$$

For convenience, we take \mathcal{B} to have zero expectation value. We write

$$\mathcal{B} = B, \quad \langle B \rangle = 0. \quad (2.4)$$

Likewise, we treat ψ as a pure fluctuation with $\langle \psi \rangle = 0$. We can then expand the Lagrangian in orders of the fluctuations, A , B , and ψ .

In zeroth order we find

$$\mathcal{L}^{(0)} = \frac{1}{2} \dot{\phi}^2 - V(\phi), \quad (2.5)$$

with the classical potential

$$V(\phi) = \frac{1}{2} \phi^2 \left(m + \frac{\lambda}{2} \phi \right)^2, \quad (2.6)$$

as sketched in Fig. 1.

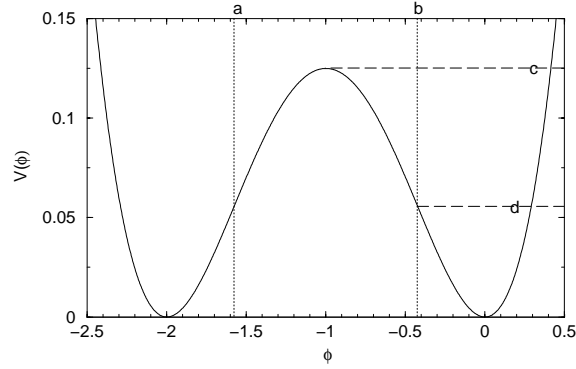


FIG. 1: The classical scalar potential with $\lambda = m = 1.0$. The dotted line represents the spinodal line above which there is sufficient energy to enter the negative curvature portion of the potential, $a = -m(1 + 1/\sqrt{3})/\lambda$, $b = -m(1 - 1/\sqrt{3})/\lambda$. The upper dashed line indicates the initial value for which the energy is high enough for ϕ to pass the maximum, $c = m^4/8\lambda$, the lower dashed line marks the upper limit for the initial value for ϕ , which leads to a stable configuration, $d = m^4/18\lambda$. Initial values in the region above d lead to unstable configurations.

This yields the classical part of the equation of motion. The first order in the fluctuation vanishes because the expectation value of the fluctuations is zero. The second order expression reads

$$\begin{aligned} \mathcal{L}^{(2)} = & \frac{1}{2} \partial_\mu A \partial^\mu A + \frac{1}{2} \partial_\mu B \partial^\mu B - \frac{m^2}{2} (A^2 + B^2) \\ & - \frac{1}{2} m \lambda \phi (3A^2 + B^2) - \frac{\lambda^2}{4} \phi^2 (3A^2 + B^2) \\ & + \frac{i}{2} \bar{\psi} \not{\partial} \psi - \frac{1}{2} m \bar{\psi} \psi - \frac{\lambda}{2} \phi \bar{\psi} \psi. \end{aligned} \quad (2.7)$$

We can derive the equation of motion for the classical field and for the fluctuations from these Lagrangians. We find for the classical field in one-loop approximation:

$$\ddot{\phi} + V'(\phi) + \frac{\lambda}{2}(m + \lambda\phi) \left[3\langle A^2 \rangle + \langle B^2 \rangle \right] + \frac{\lambda}{2}\langle \bar{\psi}\psi \rangle = 0, \quad (2.8)$$

while for the fluctuations we find

$$\ddot{A} + \left(-\nabla^2 + m^2 + \frac{3}{2}\lambda^2\phi^2 + 3m\lambda\phi \right) A = 0, \quad (2.9)$$

$$\ddot{B} + \left(-\nabla^2 + m^2 + \frac{1}{2}\lambda^2\phi^2 + m\lambda\phi \right) B = 0, \quad (2.10)$$

$$(i\partial\!\!\!/ - m - \lambda\phi)\psi = 0. \quad (2.11)$$

It is convenient to collect the mass term and the terms depending on $\phi(t)$ into time-dependent masses

$$\mathcal{M}_A^2(t) = m^2 + 3m\lambda\phi(t) + \frac{3}{2}\lambda^2\phi^2(t), \quad (2.12)$$

$$\mathcal{M}_B^2(t) = m^2 + m\lambda\phi(t) + \frac{\lambda^2}{2}\phi^2(t), \quad (2.13)$$

$$\mathcal{M}_\psi(t) = m + \lambda\phi(t). \quad (2.14)$$

We note that the supersymmetry relation

$$\mathcal{M}_A^2 + \mathcal{M}_B^2 - 2\mathcal{M}_\psi^2 = 0, \quad (2.15)$$

is satisfied by the time-dependent masses for all times. We further note that in the region

$$-1 - \frac{1}{\sqrt{3}} < \lambda\phi/m < -1 + \frac{1}{\sqrt{3}}, \quad (2.16)$$

\mathcal{M}_A^2 is negative, leading to instabilities. $\mathcal{M}_B^2(t)$ on the other hand is everywhere positive definite.

We provide integral expressions for the expectation values appearing in Eq. (2.8) below. Note that if the fermion mass $\mathcal{M}_\psi(t)$ vanishes, i.e. $\phi(t)$ is equal $-m/\lambda$, the contribution of the bosonic fields A and B cancels in (2.8). In this case, only the fermionic fluctuations influence the equation of motion and the behavior of the classical field.

We expand the scalar fields in terms of mode functions f_A and f_B , and the fermion field in terms of the spinor solutions of the Dirac equation as in [15,28]

$$A(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{A0}} \left[c_{A,\mathbf{k}} f_A(t) + c_{A,\mathbf{k}}^\dagger f_A^*(t) \right] e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (2.17)$$

$$B(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{B0}} \left[c_{B,\mathbf{k}} f_B(t) + c_{B,\mathbf{k}}^\dagger f_B^*(t) \right] e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (2.18)$$

$$\psi(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{\psi 0}} \left[b_{\mathbf{k},s} U_{\mathbf{k},s}(t) + b_{-\mathbf{k},s}^\dagger V_{-\mathbf{k},s}(t) \right] e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (2.19)$$

with the usual (anti-)commutation relations for the time independent annihilation and creation operators

$$[c_{j,\mathbf{k}}, c_{j,\mathbf{k}'}^\dagger] = 2\omega_{j0}(2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}') , \quad j = A, B , \quad (2.20)$$

$$\{b_{\mathbf{k},s}, b_{\mathbf{k}',s}^\dagger\} = 2\omega_{\psi 0}(2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}') \delta_{ss'} . \quad (2.21)$$

The frequencies in Eqs. (2.17)–(2.21) are defined as

$$\omega_{j0}^2 = k^2 + m_{j0}^2 , \quad j = A, B, \psi , \quad (2.22)$$

with $m_{j0} = \mathcal{M}_j(0)$.

The equations of motion for the bosonic mode functions are given by

$$\ddot{f}_j(t) + [k^2 + \mathcal{M}_j^2(t)] f_j(t) = 0 , \quad (2.23)$$

with $j = A, B$. The initial conditions for the fields are chosen as

$$f_j(0) = 1 , \quad \dot{f}_j(0) = -i\omega_{j0} . \quad (2.24)$$

For the fermions, we define mode functions $f_\psi(k, t)$ and $g_\psi(k, t)$ through the relations

$$U_s(\mathbf{k}, t) = N_0 [i\partial_t + \mathcal{H}_{\mathbf{k}}(t)] f_\psi(k, t) \begin{pmatrix} \chi_s \\ 0 \end{pmatrix} , \quad (2.25)$$

$$V_s(\mathbf{k}, t) = N_0 [i\partial_t + \mathcal{H}_{-\mathbf{k}}(t)] g_\psi(k, t) \begin{pmatrix} 0 \\ \chi_s \end{pmatrix} , \quad (2.26)$$

where the χ_s with $s = \pm 1$ are helicity eigenstates with eigenvalue s and $\mathcal{H}_{\mathbf{k}}(t)$ is defined as

$$\mathcal{H}_{\mathbf{k}}(t) = \gamma_0 \boldsymbol{\gamma} \cdot \mathbf{k} + \gamma_0 \mathcal{M}_\psi . \quad (2.27)$$

The mode functions $f_\psi(k, t)$ obey the second order differential equation

$$\left[\frac{d^2}{dt^2} - i\dot{\mathcal{M}}_\psi(t) + k^2 + \mathcal{M}_\psi^2(t) \right] f_\psi(k, t) = 0 , \quad (2.28)$$

while $g_\psi(k, t) = f_\psi^*(k, t)$. The initial conditions are

$$f_\psi(k, 0) = 1 , \quad \dot{f}_\psi(k, 0) = -i\omega_{\psi 0} . \quad (2.29)$$

The integrals appearing in the equation of motion for ϕ , Eq. (2.8), are:

$$\langle A^2 \rangle(t) = \int \frac{d^3 k}{(2\pi)^3} \frac{|f_A(t)|^2}{2\omega_{A0}} , \quad (2.30)$$

$$\langle B^2 \rangle(t) = \int \frac{d^3 k}{(2\pi)^3} \frac{|f_B(t)|^2}{2\omega_{B0}} , \quad (2.31)$$

$$\langle \bar{\psi}\psi \rangle(t) = - \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\omega_{\psi 0}} \left[2\omega_{\psi 0} - \frac{2k^2}{\omega_{\psi 0} + m_{\psi 0}} |f_\psi(t)|^2 \right] . \quad (2.32)$$

The energy density can be calculated as the trace over the Hamiltonian. With the results for the scalar fields in [28] and the fermion fields in [15] we can express the energy density in terms of the mode functions in the following way

$$\begin{aligned}
\mathcal{E} = & \frac{1}{2}\dot{\phi} + V[\phi(t)] \\
& + \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{A0}} \left[\frac{1}{2}|\dot{f}_A|^2 + \frac{1}{2} \left(k^2 + \mathcal{M}_A^2(t) \right) |f_A|^2 \right] \\
& + \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{B0}} \left[\frac{1}{2}|\dot{f}_B|^2 + \frac{1}{2} \left(k^2 + \mathcal{M}_B^2(t) \right) |f_B|^2 \right] \\
& + \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{\psi0}} \left[i(\omega_{\psi0} - m_{\psi0}) \left(f_{\psi} \dot{f}_{\psi}^* - \dot{f}_{\psi} f_{\psi}^* \right) - 2\omega_{\psi0} \mathcal{M}_{\psi}(t) \right] . \tag{2.33}
\end{aligned}$$

It is easy to verify that the time derivative of the energy vanishes if the equation of motion for ϕ and those of the fluctuation modes are satisfied.

Both the equation of motion for the classical field and for the energy density are divergent. Therefore we have to consider their renormalization. Within a computational scheme based on [29] and extended in [28] for non-equilibrium dynamics it is possible to make a clean separation between divergent and finite parts, allowing us to directly compute the finite parts of the integrals appearing in eqs.(2.30)-(2.32). Details of the scheme, developed for various physical models in refs. [15,28,30–35], are presented in the Appendix A.

The structure of the renormalized equations is found to be analogous to that of the unrenormalized ones, so we do not display them here. The divergent expectation values appearing in Eqs. (2.30)–(2.33) are replaced by finite, subtracted expressions. The resulting divergent counter terms are the same as the standard ones obtained in equilibrium. It is worth mentioning that supersymmetry leads to the expected cancellations of quadratic and quartic divergences leaving only the relatively well behaved logarithmic divergences – the primary advantage of supersymmetry models.

B. Results and interpretation

We find two distinct phases possible in the dynamics of the Wess-Zumino model. We examine each in turn.

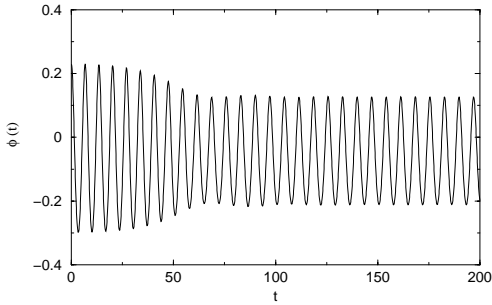


FIG. 2: Zero mode evolution showing the low energy density phase for $m = \lambda = 1.0$ and $\phi(0) = 0.23$.

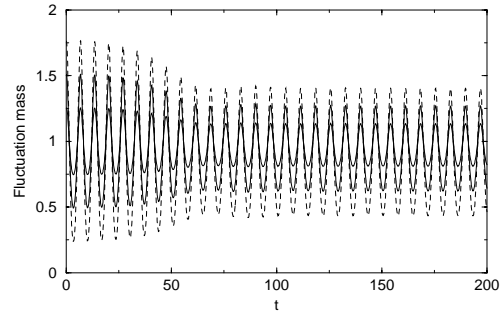


FIG. 3: The effective squared masses; solid line: $\mathcal{M}_{\psi}^2(t)$, dotted line: $\mathcal{M}_A^2(t)$, dashed line: $\mathcal{M}_B^2(t)$; the parameters as in Fig. 2.

At low energy density the model behaves much as one would expect from the scalar potential shown in Fig. 1. The zero mode ϕ oscillates about one of the two vacua, see Fig. 2. Particle production is minimal. As a result of the deviation of the zero mode from the vacuum, the masses of the various quanta each oscillate about their supersymmetric values as shown in Fig. 3. Note that while there is a small mass splitting between the masses, the overall supersymmetry sum rule, (2.15) remains satisfied. As the overall energy density is reduced to zero, these mass splittings vanish, indicating that one reaches the true supersymmetric vacuum.

These numerical results indicate that the system goes to a limiting cycle. Namely, the zero mode keeps oscillating forever. This is in contrast to the Φ^4 model in either the large N limit or in the Hartree approximation for which the zero mode always has a constant infinite time limit [26,36].

The behavior of the system is very different, however, if the initial energy density is large enough. In particular, we find that whenever the initial energy density is sufficiently large, there is an attractor solution which causes the system to fall into a state signaled dramatically by the vanishing of the fermion mass. Examination of the classical potential for ϕ (Fig. 1) and the expression for the effective mass of the fermion ψ (2.14) reveals that the only point for which the fermion is massless corresponds precisely to the local maximum of the classical potential for ϕ !

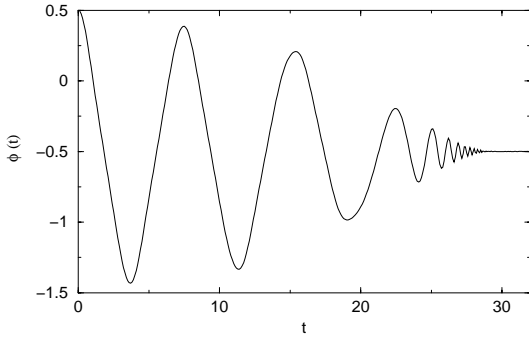


FIG. 4: Zero mode evolution showing the high energy density phase for $m = 1.0$, $\lambda = 2.0$ and $\phi(0) = 0.5$.

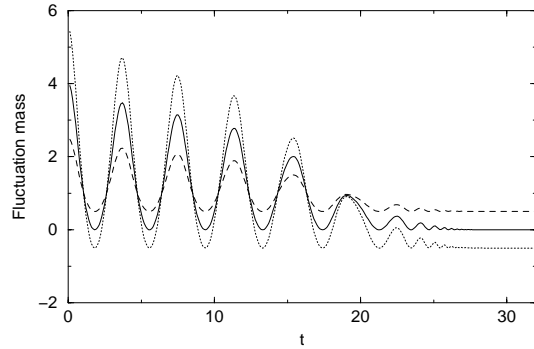


FIG. 5: The effective squared masses; solid line: $\mathcal{M}_\psi^2(t)$, dotted line: $\mathcal{M}_A^2(t)$, dashed line: $\mathcal{M}_B^2(t)$; parameters as in Fig. 4. Note that $\mathcal{M}_\psi \rightarrow 0$ while \mathcal{M}_A^2 becomes negative.

The results of a sample evolution is plotted in Fig. 4 with the effective masses of the fields given in Fig. 5. We find, indeed, that ϕ goes to the point $-m/\lambda$ with the mass of the fermion approaching zero. We also see that the evolution starts to blow up toward the end of the simulations as demonstrated in Fig. 6 (and this unstable behavior continues if allowed to evolve further). This is due simply to the fact that ϕ is sitting on top of its potential, leading to a severe instability in its fluctuations $\langle A^2 \rangle$ due to the effective negative mass squared. This is also the signal of the breakdown of the one loop approximation. In fact, it is expected that perturbation theory as a whole will break down at this point due to

the process known as spinodal decomposition [37], the result being the formation of a fully non-perturbative and inhomogeneous state.

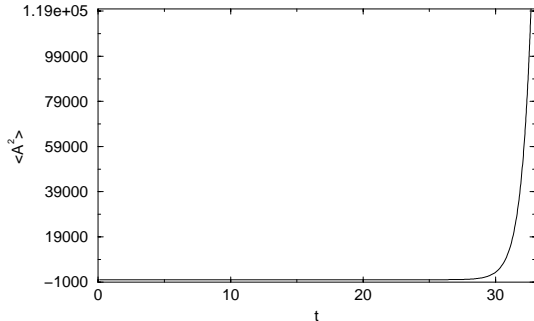


FIG. 6: The scalar fluctuation $\langle A^2 \rangle$ showing unstable behavior, parameters as in Fig. 4.

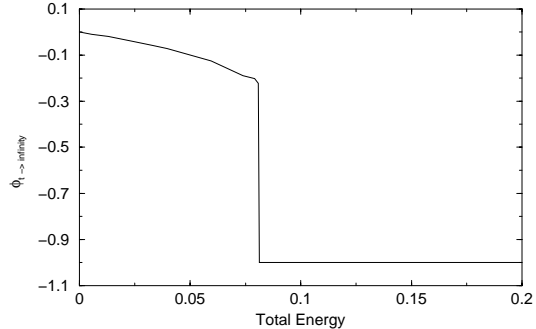


FIG. 7: Asymptotic value of ϕ as a function of the total energy with $m = \lambda = 1.0$. Notice, that $\phi_{\infty} = -m/\lambda = -1.0$ since the energy density is larger than the threshold.

Although the dynamics breaks down at one loop order, we can nevertheless consider what will happen as the evolution proceeds into the non-perturbative regime. We are helped by the sum rule (2.15) for the masses of the fields which states that $\mathcal{M}_A^2 + \mathcal{M}_B^2 = 2\mathcal{M}_{\psi}^2$ (we mention once again that this relation holds both for out of equilibrium unbroken and broken supersymmetry). First, we note that the one loop results satisfy this relation, only with the caveat that the squared mass for A is negative. Experience with non-perturbative dynamics [26,37–39] in purely scalar field theories provides clues as to what to expect. The main point is that the resulting growth of fluctuations of A due to the instability will provide a contribution tending to increase the effective mass of the field toward zero (and at the same time decreasing the B field mass toward zero). The end result will be massless fields A and B . This is, in fact, the only possible stable solution of the sum rule given massless fermions.

We therefore expect that the concave portion of the classical potential will become flattened out by non-perturbative effects due to a spinodal instability in the field A . This flattening has been seen in studies of scalar $\lambda\Phi^4$ models in the large N limit of an $O(N)$ field [38] and using the self-consistent Hartree approximation for a single scalar field [39]. These studies verified that the flattening of the potential is related to the dynamical approach to the Maxwell construction of the true free energy of the system [26,40].

It is worthwhile to discuss the processes that lead to the transition between the two dynamical regimes.

If the initial energy is sufficient for the system entering the spinodal region, Eq. (2.16), the region between a and b displayed in Fig. 1, the instability of the low momentum modes of the field \mathcal{A} comes into play. This spinodal energy density is given by $\rho_s = m^4/18\lambda^2$, marked by d in Fig. 1, and if we start the motion with $\phi(0) > 0$, the corresponding initial amplitude is $\phi_s = m(-1 + \sqrt{5/3})/\lambda$. If $\phi(0)$ exceeds ϕ_s only slightly the system enters the unstable region only for a short time, and the instability does not build up. At late times

the classical field has transferred energy to the quantum modes and no longer enters the spinodal region; it again ends up in the regime of stationary oscillation.

At higher excitations the time spent by the system in the spinodal region increases and after a few oscillations the system is trapped there. The unstable low momentum modes of the field \mathcal{A} grow indefinitely and the classical amplitude tends to $\phi_\infty = -m/\lambda$, i. e., at the maximum of the classical potential. In the phase diagram in Fig. 7 we have displayed the final value of ϕ in dependence of the total energy. The first part of the curve indicates that for low energies ϕ_∞ ends up near the minimum of the classical potential as described above. At some initial value $\phi(0) = \phi_{\text{crit}}$ we find a rather sharp transition into the unstable regime, $\phi_\infty = -m/\lambda$.

This dynamics continues for energies above the energy density of the maximum of the potential $V_{\text{max}} = m^4/8\lambda$, and to the highest excitations we have considered. Above $\rho = V_{\text{max}}$ the oscillations of the system extend into the region of the second supersymmetric minimum $\phi = -2m/\lambda$ and back again to $\phi = 0$. With each oscillation the system enters the spinodal region and the instability can build up uninhibited. The result is that the system again ends up at $\phi_\infty = -m/\lambda$. This final state is not a stationary state in the true sense, however; while the classical amplitude approaches $\phi = -m/\lambda$ the fluctuations of the field \mathcal{A} grow indefinitely, see Fig. 6.

C. Fields near the local maximum of the potential

We can analytically solve the field equations (2.8), (2.17) and (2.23) for the order parameter near the local maximum of the potential $\phi = -\frac{m}{\lambda}$.

We set,

$$\phi(t) = -\frac{m}{\lambda} + \Delta(t) \quad (2.34)$$

and we will restrict to times where $\Delta(t) \ll m$.

Eqs.(2.23) for $f_A(t)$ thus becomes

$$\ddot{f}_A(t) + \left[k^2 - \frac{m^2}{2} + \frac{3}{2} \lambda^2 \Delta^2(t) \right] f_A(t) = 0$$

showing that the A -modes with $k < m/\sqrt{2}$ are growing exponentially as $e^{t\sqrt{\frac{m^2}{2}-k^2}}$. Therefore, the quantum fluctuations $\langle A^2 \rangle$ [see eq.(2.30)] will be dominated by these spinodally unstable low- k modes and will grow as

$$\langle A^2 \rangle(t) = \int \frac{d^3k}{(2\pi)^3} \frac{|f_A(t)|^2}{2\omega_{A0}} \stackrel{mt \gtrsim 1}{\approx} \frac{e^{\sqrt{2}mt}}{(mt)^{3/2}} D(t),$$

where the function $D(t)$ is the order one for late times. The equation of motion for the zero mode (2.8) takes the form

$$\ddot{\Delta}(t) + \left\{ -\frac{m^2}{2} + \frac{1}{2} \lambda^2 [\Delta^2(t) + 3\langle A^2 \rangle(t) + \langle B^2 \rangle(t)] \right\} \Delta(t) + \frac{1}{2} \lambda \langle \bar{\psi}\psi \rangle(t) = 0. \quad (2.35)$$

Since as t grows the zero mode approaches $-\frac{m}{\lambda}$, $\Delta(t) \rightarrow 0$ and $\langle A^2 \rangle(t) \rightarrow \infty$. Therefore, in order eq. (2.35) keep valid, the product

$$\langle A^2 \rangle(t) \Delta(t) = G(t) ,$$

must stay bounded. Moreover, it must fulfill

$$G(t) = -\frac{1}{3\lambda} \langle \bar{\psi}\psi \rangle(t) , \quad (2.36)$$

a relation which is satisfied by the dynamics in the numerical simulations, see Fig. 8.

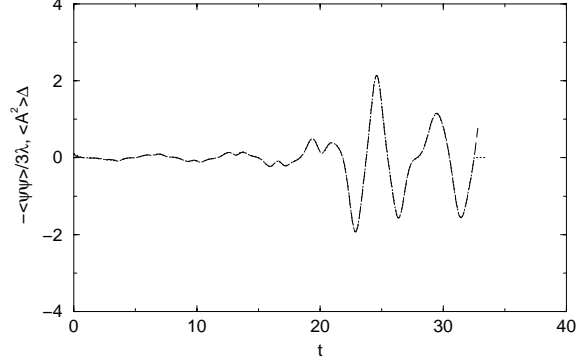


FIG. 8: Numerical check of $\langle A^2 \rangle \Delta = -\langle \bar{\psi}\psi \rangle / 3\lambda$, dashed line: $\langle A^2 \rangle \Delta$, dotted line: $-\langle \bar{\psi}\psi \rangle / 3\lambda$.

III. LARGE N EXTENSION OF THE WESS-ZUMINO MODEL

Having seen that the self-consistent one-loop approximation to the Wess-Zumino model breaks down due to the spinodal instability, we now turn to an extension of the model for which we can follow the evolution into the non-perturbative domain. We couple the Wess-Zumino superfield to N chiral superfields satisfying an internal $O(N)$ symmetry. By taking the large N limit, we arrive at a semi-classical supersymmetric model for which we can numerically solve for the exact and fully non-perturbative field dynamics.

A. Model and equations of motion

Our $O(N)$ extension of the Wess-Zumino model consists of a chiral superfield multiplet $S_0 = (A_0, B_0; \psi_0, F_0, G_0)$, which acts as a singlet under $O(N)$, coupled to N chiral superfields $S_i = (A_i, B_i; \psi_i, F_i, G_i)$ with $i = 1 \dots N$ and which transform as a vector under $O(N)$. The superpotential has the form

$$\mathcal{L} = \frac{1}{2} M S_0 \cdot S_0 + \frac{\kappa}{6\sqrt{N}} S_0 \cdot S_0 \cdot S_0 + \frac{1}{2} m \sum_{i=1}^N S_i \cdot S_i + \frac{\lambda}{2\sqrt{N}} \sum_{i=1}^N S_0 \cdot S_i \cdot S_i . \quad (3.1)$$

Notice that the model considered in Ref. [41] is a special case of ours for $m = \kappa = 0$. Again, we expand in terms of the component fields and eliminate the auxiliary fields via their equations of motion. To allow for a consistent large N limit, the expectation value of A_0 is taken to be of order \sqrt{N} . We set

$$\langle A_0 \rangle = \sqrt{N}\phi, \quad \langle B_0 \rangle = 0. \quad (3.2)$$

The latter condition, which amounts to a choice of initial conditions, is chosen because it significantly simplifies the equations of motion. We assume that the initial state satisfies the $O(N)$ symmetry which requires that $\langle A_i \rangle = \langle B_i \rangle = 0$. Given the $O(N)$ symmetry, it is convenient to define fields A , B , and ψ such that $\sum_i A_i A_i = NA^2$, $\sum_i B_i B_i = NB^2$, and $\sum_i \bar{\psi}_i \psi_i = N\bar{\psi}\psi$. In taking the large N limit, particular care must be given to terms of the form $\sum_i A_i B_i$ which turn out to be of order \sqrt{N} . This is most easily seen by computing the squared quantity $\sum_i \sum_j A_i B_i A_j B_j$; the resulting delta function δ_i^j contributes a factor $1/N$ which cancels one of the factors of N coming from the summations. The Lagrangian to leading order in N is

$$\begin{aligned} \frac{\mathcal{L}}{N} = & \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu A \partial^\mu A + \frac{1}{2} \partial_\mu B \partial^\mu B - \frac{1}{2} \phi^2 \left(M + \frac{1}{2} \kappa \phi \right)^2 \\ & - \frac{1}{2} (m + \lambda \phi)^2 (A^2 + B^2) - \frac{\lambda}{2} \left[M\phi + \frac{1}{2} \kappa \phi^2 + \frac{1}{4} \lambda (A^2 - B^2) \right] (A^2 - B^2) \\ & + \frac{i}{2} \bar{\psi} \not{\partial} \psi - \frac{1}{2} m \bar{\psi} \psi - \frac{1}{2} \lambda \phi \bar{\psi} \psi. \end{aligned} \quad (3.3)$$

Note that the fluctuations of the supersymmetry multiplet S_0 do not appear to this order in N , with only the mean field ϕ contributing to the Lagrangian. The consequence is that the Lagrangian of this theory is equivalent to a supersymmetric $O(N)$ field S_i coupled to a fully classical background zero mode ϕ . This is a consequence of the semi-classical nature of the large N approximation.

The equation of motion for the mean field ϕ is found to be

$$\begin{aligned} \ddot{\phi} + (M + \kappa \phi) \left[M\phi + \frac{1}{2} \kappa \phi^2 + \frac{1}{2} \lambda \langle A^2 - B^2 \rangle \right] \\ + \lambda (m + \lambda \phi) \langle A^2 + B^2 \rangle + \frac{1}{2} \lambda \langle \bar{\psi} \psi \rangle = 0, \end{aligned} \quad (3.4)$$

while the fermion obeys

$$[i\not{\partial} - (m + \lambda \phi)] \psi = 0, \quad (3.5)$$

and the mode functions from which the scalar fluctuations are built satisfy

$$\ddot{f}_A + k^2 f_A + (m + \lambda \phi)^2 f_A + \lambda \left[M\phi + \frac{1}{2} \kappa \phi^2 + \frac{1}{2} \lambda \langle A^2 - B^2 \rangle \right] f_A = 0, \quad (3.6)$$

$$\ddot{f}_B + k^2 f_B + (m + \lambda \phi)^2 f_B - \lambda \left[M\phi + \frac{1}{2} \kappa \phi^2 + \frac{1}{2} \lambda \langle A^2 - B^2 \rangle \right] f_B = 0. \quad (3.7)$$

The integrals for the fluctuations are constructed just as in the ordinary Wess-Zumino model via Eqs. (2.30)–(2.32). It is convenient to define the masses

$$\mathcal{M}_\psi \equiv m + \lambda\phi, \quad (3.8)$$

$$\mathcal{M}_-^2 \equiv \lambda \left[M\phi + \frac{1}{2}\kappa\phi^2 + \frac{1}{2}\lambda\langle A^2 - B^2 \rangle \right], \quad (3.9)$$

such that

$$\mathcal{M}_A^2 = \mathcal{M}_\psi^2 + \mathcal{M}_-^2, \quad (3.10)$$

$$\mathcal{M}_B^2 = \mathcal{M}_\psi^2 - \mathcal{M}_-^2. \quad (3.11)$$

We see again that the supersymmetry sum rule $\mathcal{M}_A^2 + \mathcal{M}_B^2 - 2\mathcal{M}_\psi^2 = 0$ is automatically satisfied.

The equation for \mathcal{M}_- , (3.9), plays the role of a gap equation expected in the large N framework, since the fluctuations appearing in the integrals on the right hand side depend upon the masses \mathcal{M}_A and \mathcal{M}_B which in turn depend upon \mathcal{M}_- . This becomes more explicit in the fully renormalized form found in Appendix B. We also note that \mathcal{M}_-^2 can be either positive or negative, with the consequence that either or both \mathcal{M}_A^2 and \mathcal{M}_B^2 can become negative during the evolution.

The energy density is

$$\begin{aligned} \mathcal{E} = & \frac{1}{2}\dot{\phi}^2 + \frac{1}{2} \left(M\phi + \frac{\kappa}{2}\phi^2 \right)^2 \\ & + \frac{1}{2} \langle \dot{A}^2 + k^2 A^2 + \mathcal{M}_A^2(t) A^2 \rangle + \frac{1}{2} \langle \dot{B}^2 + k^2 B^2 + \mathcal{M}_B^2(t) B^2 \rangle \\ & - \frac{\lambda^2}{8} \left(\langle A^2 - B^2 \rangle \right)^2 + \frac{1}{2} \langle \bar{\psi} \left(-i\vec{\gamma} \cdot \vec{\nabla} + \mathcal{M}_\psi(t) \right) \psi \rangle. \end{aligned} \quad (3.12)$$

Again the equations of motion and the energy density have to be renormalized. Despite the non-perturbative nature of the large N limit, one of its important properties is that it is possible to consistently renormalize the theory. The details are presented in Appendix B. We separate out the divergent terms so that they may be treated analytically, leaving the finite parts to be included in numerical simulations. This allows us the freedom to choose a regularization scheme without regard to constraints of the numerical simulations. We then use dimensional regularization to define the counterterms. Again the structure of the finite equations remains essentially the same as the one of the bare equations, with divergent expectation values replaced by finite ones, and with finite corrections to the masses and couplings. We find consistency with the renormalization of the large- N equilibrium theory.

B. The supersymmetry in the large N limit

Examining the leading order Lagrangian, Eq. (3.3), the question arises as to whether taking the large N limit is consistent with supersymmetry. One might be particularly concerned since the singlet superfield S_0 appearing in (3.1) is represented only by the single scalar zero mode $\phi = \langle A_0 \rangle / \sqrt{N}$ and one might expect that it would be necessary to have a corresponding fermion field into which ϕ may transform. To clarify this point we examine the transformation properties of A_0 . Under supersymmetry, A_0 transforms as

$$A'_0 = A_0 + \delta A_0 = A_0 + \bar{\zeta} \psi_0, \quad (3.13)$$

where ζ is the Grassman supersymmetry transformation parameter. Taking the expectation value of Eq. (3.13) yields the transformation law for ϕ :

$$\phi' \equiv \langle A_0' \rangle / \sqrt{N} = \langle A_0 \rangle / \sqrt{N} + \langle \bar{\zeta} \psi_0 \rangle / \sqrt{N} = \phi + \bar{\zeta} \langle \psi_0 \rangle / \sqrt{N} = \phi , \quad (3.14)$$

where the last equality follows from the vanishing expectation value of ψ_0 . We therefore see that ϕ is invariant under supersymmetry transformations. This is consistent with the treatment of ϕ as a classical background zero mode coupled to the supersymmetric $O(N)$ multiplet.

To leading order, it is no longer necessary to consider the transformation properties of the $O(N)$ singlet fields corresponding to the superfield S_0 as none of these fields appear in our large N Lagrangian (3.3). The remaining transformations are:

$$\delta\phi = 0 , \quad (3.15)$$

$$\delta A = \bar{\zeta} \psi , \quad (3.16)$$

$$\delta B = \bar{\zeta} \gamma_5 \psi , \quad (3.17)$$

$$\delta\psi = (i\not{\partial} + m + \lambda\phi) (A + \gamma_5 B) \zeta , \quad (3.18)$$

$$\delta\bar{\psi} = \bar{\zeta} [-i\not{\partial} (A - \gamma_5 B) + (m + \lambda\phi) (A + \gamma_5 B)] . \quad (3.19)$$

Through use of the equations of motion for A , B , and ψ , it is straightforward to show that the variation of the Lagrangian (3.3) vanishes under these supersymmetry transformations up to a total derivative. Therefore, we see that to this order, the Lagrangian is completely supersymmetric with ϕ acting as a classical background field.

C. Results and interpretation

As in the ordinary Wess-Zumino model, we find two distinct phases, one at low energy densities and one at high energy densities.

Unlike the ordinary Wess-Zumino model, however, this model has perturbative vacua for which the fermion masses vanish.

In the high energy density phase, the time evolution leads to a final state formed by a cloud of massless particles (both bosons and fermions) around a minimum $\phi = -m/\lambda$ of the effective potential, degenerate with the tree level minimum. This nonperturbative minimum defines a new sector of the theory. [The perturbative sector is formed by particles around the perturbative ground state $\phi = 0$.] Since this nonperturbative minimum at $\phi = -m/\lambda$ is degenerate with the perturbative minimum, it is invariant under supersymmetry. Hence, supersymmetry is not broken at this nonperturbative minimum. In addition, the fact that all masses vanish at the new minimum $\phi = -m/\lambda$ supports the supersymmetric character of this point. In addition, we choose $O(N)$ invariant states since $\langle A_i \rangle = \langle B_i \rangle = 0$ for all times.

However, the final state obtained by real time evolution for large energy density is formed by a cloud of massless particles (both bosons and fermions) around this new nonperturbative minimum $\phi = -m/\lambda$. This state indeed breaks supersymmetry since the energy is distributed differently among the fermions and bosons on the top of this zero-energy nonperturbative ground state due to their differing statistics. This is analogous to which is

known from equilibrium studies at finite temperature where a thermal gas of particles is around a perturbative supersymmetric vacuum.

In summary, for large energy density the system goes to a nonperturbative minimum at $\phi = -m/\lambda$ which acts as an attractor to the dynamics and where a kind of Goldstone phase develops with all particles, bosons and fermions, being massless. Interestingly, these phenomena happens for the highest energy densities tested. The reason that keeps all particles massless is that supersymmetry results in cancellations in the contributions to the effective masses of the fields, see eqs.(3.9) – (3.11).

The situation here looks similar to the case where there is no symmetry restoration at high energies and massless Goldstone bosons are therefore present [42,43]. While individual field fluctuations may become large as the energy is increased, the net contribution of the fluctuations does not grow. Thus, arbitrarily high energy densities need not yield the mass corrections which ordinarily lead to symmetry restoration in spontaneously broken $O(N)$ -theories [26].

We now examine each phase in detail.

The low energy, massive phase occurs in much the same way as in the ordinary Wess-Zumino model and is depicted in Fig. 9. with the corresponding masses shown in Fig. 10. However, because of the more complicated structure of the potential in the large N model, the range of energy densities for which the low energy density phase persists depend on the initial conditions. One can distinguish two cases:

1. $\phi_0 > -M/\kappa$ and $m/\lambda < M/\kappa$, or $\phi_0 < -M/\kappa$ and $m/\lambda > M/\kappa$. In either of these two cases, the zero mode ϕ begins on the same side of its potential as the new nonperturbative supersymmetric minimum. This results in a relatively low critical energy density for reaching the high energy density phase.
2. $\phi_0 > -M/\kappa$ and $m/\lambda > M/\kappa$, or $\phi_0 < -M/\kappa$ and $m/\lambda < M/\kappa$. In these cases, there is an additional potential barrier between the initial value of ϕ and the new nonperturbative supersymmetric minimum. As a result, the transition to the high energy density phase occurs at a higher critical value of the energy density.

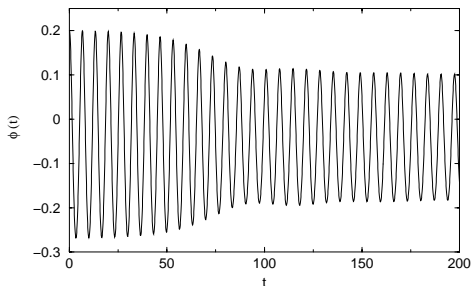


FIG. 9: Zero mode evolution showing the low energy density phase for $m = M = 1.0$, $\kappa = \lambda = 1.0$ and $\phi(0) = 0.2$.

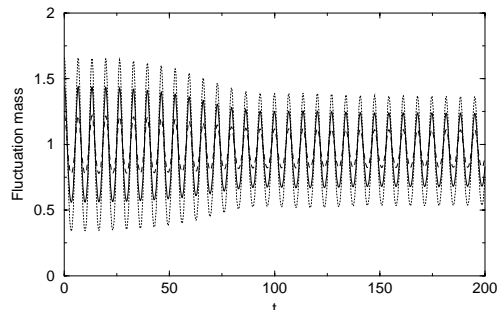


FIG. 10: The effective squared masses; solid line: $\mathcal{M}_\psi^2(t)$, dotted line: $\mathcal{M}_A^2(t)$, dashed line: $\mathcal{M}_B^2(t)$; the parameters as in Fig. 9.

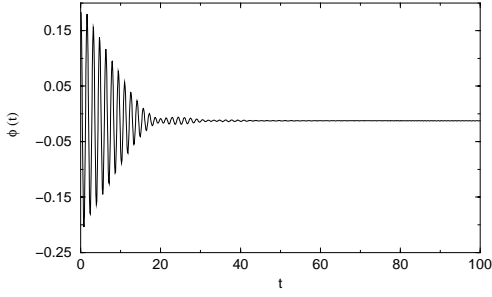


FIG. 11: Zero mode evolution showing the low energy density phase for $m = 1.0$, $M = 4.0$, $\kappa = \lambda = 1.0$ and $\phi(0) = 0.2$.

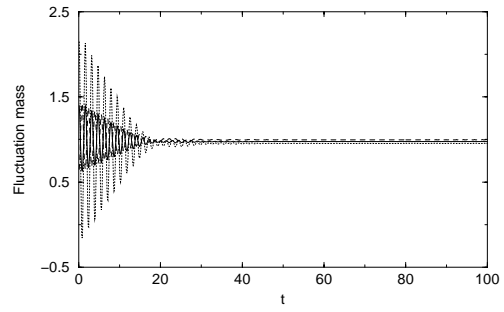


FIG. 12: The effective squared masses; solid line: $\mathcal{M}_\psi^2(t)$, dotted line: $\mathcal{M}_A^2(t)$, dashed line: $\mathcal{M}_B^2(t)$; parameters as in Fig. 11.

A second example of low energy density evolution is shown in Figs. 11 and 12 for which the masses are such that ϕ may efficiently decay into A and B particles, leading to apparent dissipation. We see that ϕ decays and settles at a point near the classical minimum at $\phi = 0$. The shift of the finite density minimum from this vacuum value is due to the growth of the fluctuations of the fields A , B , and ψ , which are also responsible for the deviation of the masses from the supersymmetry value m .

An example of the high energy density phase is depicted in Fig. 13 with the masses in Fig. 14. The evolution begins with large oscillations of ϕ over the entire classically allowed range of evolution. During this initial period, the field fluctuations $\langle A^2 \rangle$ and $\langle B^2 \rangle$ grow. After a relatively short period of time, the mean field settles down precisely to the point $\phi = -m/\lambda$. The result is that the N fermions become massless. As in the one-loop Wess Zumino case, this is an attractor state at high energy densities. However, in the present model the massless state is stable. A look to the effective potential [20] shows that $\phi = -m/\lambda$ is a zero-energy minimum of the effective potential. This minimum is degenerate with the perturbative vacuum $\phi = 0$.

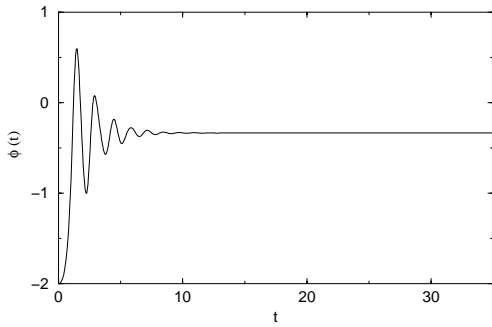


FIG. 13: Zero mode evolution showing the high energy density phase for $m = 1.0$, $M = 4.0$, $\kappa = 2.0$, $\lambda = 3.0$ and $\phi(0) = -1.5$.

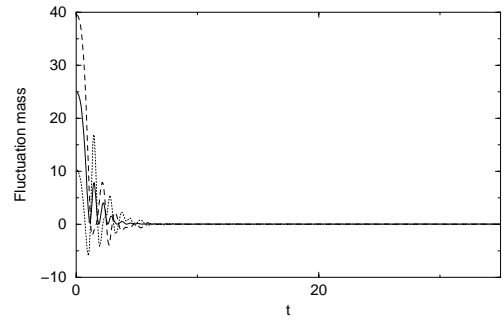


FIG. 14: The effective squared masses; solid line: $\mathcal{M}_\psi^2(t)$, dotted line: $\mathcal{M}_A^2(t)$, dashed line: $\mathcal{M}_B^2(t)$; parameters as in Fig. 13.

We therefore find that the system reaches a non-perturbative state for which each of the $O(N)$ fermion and boson fields is massless and for which the vacuum energy identically vanishes as well indicating the presence of a new non-perturbative supersymmetric minimum at $\phi = -m/\lambda$. The fact that the massless state is an attractor is of great importance if one considers the model from a cosmological perspective. What it indicates is that the initial evolution of the system in the early universe can determine the ultimate vacuum state of the system, providing an effective means of selection between vacua which are otherwise degenerate in energy.

IV. ADDING SOFT BREAKING TERMS

One question that might be raised is what are the consequences of adding additional mass terms to the model which softly break the supersymmetry. This is particularly interesting because, as we have seen, the dynamics leads to massless fermions and, in the case of the $O(N)$ model, massless bosons as well. Through soft supersymmetry breaking, it may be possible to introduce small masses to these final state particles, which would be determined not by the scale of supersymmetry, rather by some lower scale (e.g., the electroweak scale) at which the soft supersymmetry breaking terms arise.

To provide an example case, we introduce soft supersymmetry breaking to the $O(N)$ model via a scalar mass m_s for the A and B fields such that Eqs. (3.10) and (3.11) become $\mathcal{M}_A^2 = \mathcal{M}_\psi^2 + \mathcal{M}_-^2 + m_s^2$ and $\mathcal{M}_B^2 = \mathcal{M}_\psi^2 - \mathcal{M}_-^2 + m_s^2$. Such terms break the supersymmetry explicitly, while not producing any dangerous (i.e. non-logarithmic) divergences. We plot the results for the case $m_s/m = 0.1$ in Fig. 15.

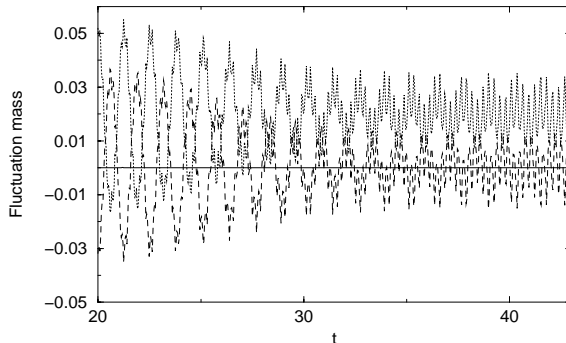


FIG. 15: The effective field masses in the high energy density phase including soft breaking masses for the fields A and B ; solid line: $\mathcal{M}_\psi^2(t)$, dotted line: $\mathcal{M}_A^2(t)$, dashed line: $\mathcal{M}_B^2(t)$; the parameters are $m = 1.0$, $M = 4.0$, $m_s = 0.1$, $\kappa = 2.0$, $\lambda = 3.0$, $\phi(0) = 0.9$.

The first thing to note is that we retain the high energy density attractor solution. As the new terms do not explicitly break the $O(N)$ symmetry, there is necessarily one set of asymptotically massless Goldstone bosons, in this case represented by the B field. However, these fields' superpartners are no longer massless as they pick up contributions proportional to the soft supersymmetry breaking mass scale m_s . For A , this mass is given approximately by $\mathcal{M}_A \simeq \sqrt{2}m_s$, corresponding to $\mathcal{M}_- \simeq m_s$ and $\mathcal{M}_\psi \simeq 0$. For the fermions, there is a see-

saw mechanism producing the mass such that the leading order contribution is proportional to m_s^2/m . This produces fermion masses which are suppressed by a factor of m_s/m relative to their massive bosonic superpartners.

V. DISCUSSION AND CONCLUSIONS

Let us first summarize our main results.

1. At low energy density, both the one loop and the large N Wess-Zumino models are found to allow for finite density non-equilibrium states based on supersymmetric vacua. Finite density corrections break the mass degeneracy of the scalars and fermions somewhat, but do not lead to obvious massless fermions. In the large N model, such states are stable, albeit time dependent. They are also stable in the ordinary Wess-Zumino model to one loop order.
2. At high energy density, the dynamics of the Wess-Zumino model and its large N extension leads to massless fermions. These states are stable in the large N model, but are highly unstable at one loop order in the ordinary Wess-Zumino model.
3. In the large N model, the high energy density phase is characterized for arbitrarily high energy densities by all of the $O(N)$ vector fields, scalars and fermions, becoming massless, and by the vanishing of the total vacuum energy. The phase is formed by a cloud of massless particles (both bosons and fermions) around a new nonperturbative supersymmetric minimum at $\phi = -m/\lambda$.
4. The introduction of explicit soft supersymmetry breaking terms to the large N Lagrangian results in masses for some of the $O(N)$ vector fields. In particular, we find that a soft supersymmetry mass of m_s for the scalars induces a mass for the A field $\mathcal{M}_A = \sqrt{2}m_s$ and a fermion mass of order m_s^2/m , while B remains massless.

We can visualize a number of consequences for cosmology and particle physics.

1. It appears that we should expect massless fermions to occur in supersymmetric models of the early universe independent of whether the situation is one of equilibrium or far from equilibrium, or at least, if the universe is far from equilibrium, we should expect it to approach such a state.
2. If the vacuum energy is indeed zero as in the large N case, then we have a convenient mechanism by which otherwise massive fermions (and possibly other fields as well) become massless. This could have the effect of priming the system so that soft supersymmetry breaking terms can yield fields with masses much smaller than the overall scale of supersymmetry.
3. A final consequence of potential interest is that supersymmetry may protect continuous symmetries from being restored in the very early universe, as in our $O(N)$ model. Such a result might alleviate the monopole problem, as without a symmetry restored phase, there would be no phase transition to produce such objects.

We have only begun to examine these possibilities, but many of these ideas can be pursued further using current techniques in out of equilibrium quantum field theory. One avenue of approach is to examine the ordinary Wess-Zumino beyond one loop order by means of mean field theory. This may clarify what happens to the masses of the A and B fields as well as having the potential to determine the vacuum energy of the resulting non-perturbative state. Also of interest is the behavior of these models in an expanding universe, as it would be important to see if a phase transition from the high energy phase to the low energy density phase occurs when energy is drained from the system via expansion. The relation between the expansion time scale and the relaxation time scale in the high energy density phase may also play a significant role.

Of course, it would be beneficial to examine the detailed behavior of gauge multiplets in order to begin to get a better understanding of more realistic models of particle physics. It would also be very interesting to examine the possibility that neutrino masses could be realistically generated via an analogous mechanism to that of fermion mass generation in the softly broken supersymmetry model studied here. The techniques are available to tackle such problems, and the results of the present study encourage us to believe that there is much more to be learned from further studies of the out of equilibrium dynamics of supersymmetric particle physics.

ACKNOWLEDGMENTS

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APPENDIX A: RENORMALIZATION: THE WESS ZUMINO MODEL

In order to renormalize the one-loop-equations we use the perturbation scheme of [28] which allows us to extract the divergences from the leading orders. We recall some basic equations and refer the interested reader to [15,28] for further details. We rewrite the mode equations (2.23) and (2.28) as

$$\left[\frac{d^2}{dt^2} + \omega_{j0}^2 \right] f_j(t) = -\mathcal{V}_j(t) f_j(t) , \quad (\text{A1})$$

with $j = A, B, \psi$ and

$$\mathcal{V}_A(t) = \frac{3}{2}\lambda^2 (\phi^2 - \phi_0^2) + 3m\lambda (\phi - \phi_0) = \mathcal{M}_A^2(t) - m_{A0}^2 , \quad (\text{A2})$$

$$\mathcal{V}_B(t) = \frac{1}{2}\lambda^2 (\phi^2 - \phi_0^2) + m\lambda (\phi - \phi_0) = \mathcal{M}_B^2(t) - m_{B0}^2 , \quad (\text{A3})$$

$$\begin{aligned} \mathcal{V}_\psi(t) &= \lambda^2 (\phi^2 - \phi_0^2) + 2\lambda m (\phi - \phi_0) - i\lambda \dot{\phi} \\ &= \mathcal{M}_\psi^2(t) - m_{\psi 0}^2 - i\dot{\mathcal{M}}_\psi(t) . \end{aligned} \quad (\text{A4})$$

With the initial conditions (2.24) it is possible to write the differential equation (A1) as an equivalent integral equation

$$f_j(t) = e^{-i\omega_{j0}t} - \frac{1}{\omega_{j0}} \int_0^t dt' \sin[\omega_{j0}(t-t')] \mathcal{V}_j(t') f_j(t') . \quad (\text{A5})$$

We can now make a general ansatz for the mode functions via

$$f_j(t) = e^{-i\omega_{j0}t} [1 + h_j(t)] . \quad (\text{A6})$$

Inserting this ansatz into the integral equation one derives an iteration that results in an expansion of the functions $h_j(t)$ in orders of the potentials $\mathcal{V}_j(t)$.

The perturbative expansion for the bosonic fluctuation integrals leads to [28]

$$\langle A^2 \rangle = I_{-1}(m_{A0}) - \mathcal{V}_A(t) I_{-3}(m_{A0}) + \langle A^2 \rangle_{\text{fin}} , \quad (\text{A7})$$

$$\langle B^2 \rangle = I_{-1}(m_{B0}) - \mathcal{V}_B(t) I_{-3}(m_{B0}) + \langle B^2 \rangle_{\text{fin}} . \quad (\text{A8})$$

The integrals I_{-k} , $k = 1, 3$ are divergent. In dimensional regularization they become

$$I_{-3}(m_{j0}^2) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{4\omega_{j0}^3} \rightarrow \frac{1}{16\pi^2} \left(\frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{m_{j0}^2} - \gamma \right) , \quad (\text{A9})$$

$$I_{-1}(m_{j0}^2) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{j0}} \rightarrow -m_{j0}^2 I_{-3}(m_{j0}^2) - \frac{m_{j0}^2}{16\pi^2} . \quad (\text{A10})$$

The finite parts are defined as

$$\langle A^2 \rangle_{\text{fin}} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{A0}} \left\{ |f_A(t)|^2 - 1 + \frac{1}{2\omega_{A0}^2} \mathcal{V}_A(t) \right\} , \quad (\text{A11})$$

$$\langle B^2 \rangle_{\text{fin}} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{B0}} \left\{ |f_B(t)|^2 - 1 + \frac{1}{2\omega_{B0}^2} \mathcal{V}_B(t) \right\} . \quad (\text{A12})$$

The integration over the subtracted integrand is finite. Note that using the ansatz (A6) and working with the functions $h_j(t)$ the leading divergence cancels explicitly, i.e., it is not included from the outset.

For the fermions we obtain

$$\langle \bar{\psi}\psi \rangle = I_{-3}(m^2) [2\ddot{\mathcal{M}}_\psi + 4\mathcal{M}_\psi^3] + \mathcal{F}_{\text{fin}}^\psi , \quad (\text{A13})$$

with

$$\mathcal{F}_{\text{fin}}^\psi(t) = \frac{1}{16\pi^2} \left\{ 4m_{\psi0}^2 \mathcal{M}_\psi(t) - \ln \left(\frac{m_{\psi0}^2}{m^2} \right) [4\mathcal{M}_\psi^3(t) + 2\ddot{\mathcal{M}}_\psi(t)] \right\} + \langle \bar{\psi}\psi \rangle_{\text{fin}} . \quad (\text{A14})$$

It should be mentioned that the finite parts as defined here generally become *singular* as $t \rightarrow 0$. These initial singularities can be removed by a Bogoliubov transformation [44]. We do not discuss this here, as it is not essential in the present context.

The total contribution of the fluctuations in the equation of motion (2.8) has the form

$$\mathcal{F} = \frac{1}{2} \lambda^2 \phi [3\langle A^2 \rangle + \langle B^2 \rangle] + \frac{m\lambda}{2} \left[3\langle A^2 \rangle + \langle B^2 \rangle + \frac{1}{m} \langle \bar{\psi}\psi \rangle \right] . \quad (\text{A15})$$

Inserting the expressions for the expectation values given above we find

$$\begin{aligned}\mathcal{F} = & \lambda^2 \left(\ddot{\phi} - m^2 - \frac{3}{2}\lambda m\phi^2 - \frac{1}{2}\lambda^2\phi^3 \right) I_{-3}(m) \\ & + \Delta Z \ddot{\phi} + \Delta F + \Delta m\phi + \Delta\lambda \frac{3}{2}m\lambda\phi^2 + \frac{1}{2}\Delta\lambda\lambda^2\phi^3 + \mathcal{F}_{\text{fin}} ,\end{aligned}\tag{A16}$$

where \mathcal{F}_{fin} is defined by (A15), with all the expectation values replaced by their finite parts. Note that instead of the divergent integrals $I_{-k}(m_{i0})$ with the different initial masses one has been able to collect the divergent part into one integral $I_{-3}(m)$ where m is the renormalization mass, chosen here as the common physical mass parameter of the model. The dependence on the initial conditions appears in the finite terms

$$\Delta Z = \frac{\lambda^2}{8\pi^2} \ln \frac{m_{\psi 0}^2}{m^2} ,\tag{A17}$$

$$\begin{aligned}\Delta F = & -\frac{1}{32\pi^2} m\lambda (3m_{A0}^2 + m_{B0}^2 - 4m_{\psi 0}^2) \\ & -\frac{m^3\lambda}{32\pi^2} \left(3 \ln \frac{m_{A0}^2}{m^2} + \ln \frac{m_{B0}^2}{m^2} - 4 \ln \frac{m_{\psi 0}^2}{m^2} \right) ,\end{aligned}\tag{A18}$$

$$\begin{aligned}\Delta m = & -\frac{\lambda^2}{32\pi^2} (3m_{A0}^2 + m_{B0}^2 - 4m_{\psi 0}^2) \\ & -\frac{\lambda^2 m^2}{16\pi^2} \left(6 \ln \frac{m_{A0}^2}{m^2} + \ln \frac{m_{B0}^2}{m^2} - 6 \ln \frac{m_{\psi 0}^2}{m^2} \right) ,\end{aligned}\tag{A19}$$

$$\Delta\lambda = -\frac{\lambda^2}{32\pi^2} \left(9 \ln \frac{m_{A0}^2}{m^2} + \ln \frac{m_{B0}^2}{m^2} - 8 \ln \frac{m_{\psi 0}^2}{m^2} \right) .\tag{A20}$$

The divergent part of (A16) can be written in the form

$$\mathcal{F}_{\text{inf}} \lambda^2 [\ddot{\phi} - V'(\phi)] I_{-3}(m) .\tag{A21}$$

It can be removed by adding to (2.1) a counter term superlagrangian

$$\mathcal{L}_{\text{c.t.}} = \left(\frac{1}{2} \delta Z S \cdot TS - \frac{1}{2} \delta m S \cdot S - \frac{1}{3} \delta \lambda S \cdot S \cdot S \right)_F ,\tag{A22}$$

with

$$\delta Z = -\lambda^2 I_{-3}(m) ,\tag{A23}$$

$$\frac{\delta m}{m} = \frac{\lambda^2}{2} I_{-3}(m) ,\tag{A24}$$

$$\frac{\delta \lambda}{\lambda} = \frac{\lambda^2}{2} I_{-3}(m) .\tag{A25}$$

The equivalence between the mass and the coupling constant counter term is a special feature of supersymmetry. With these counter terms taken into account the equation of motion becomes finite. Explicitly it is given by

$$\begin{aligned}(1 + \Delta Z) \ddot{\phi} + (m^2 + \Delta m)\phi + \frac{3}{2}m\lambda(1 + \Delta\lambda)\phi^2 \\ + \frac{1}{2}\lambda^2(1 + \Delta\lambda)\phi^3 + \Delta F + \mathcal{F}_{\text{fin}} = 0 .\end{aligned}\tag{A26}$$

In the same way we have split \mathcal{F} into divergent and finite parts we can decompose the fluctuation part of the energy as

$$\mathcal{E}_{\text{fl}}(t) = \mathcal{E}_{\text{fl,div}}(t) + \mathcal{E}_{\text{fl,fin}}(t) , \quad (\text{A27})$$

with

$$\begin{aligned} \mathcal{E}_{\text{fl,div}} = & \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{2} (\omega_{A0} + \omega_{B0} - 2\omega_{\psi0}) \right. \\ & + \frac{1}{4} \left[\frac{\mathcal{V}_A}{\omega_{A0}} + \frac{\mathcal{V}_B}{\omega_{B0}} - \frac{2}{\omega_{\psi0}} (\mathcal{M}_\psi^2(t) - m_{\psi0}^2) \right] \\ & \left. - \frac{1}{16} \left[\frac{\mathcal{V}_A^2}{\omega_{A0}^3} + \frac{\mathcal{V}_B^2}{\omega_{B0}^3} - \frac{2}{\omega_{\psi0}^3} (\dot{\mathcal{M}}_\psi^2(t) + \mathcal{M}_\psi^4(t) + m_{\psi0}^4 - 2\mathcal{M}_\psi^2(t)m_{\psi0}^2) \right] \right\} , \quad (\text{A28}) \end{aligned}$$

$$\begin{aligned} \mathcal{E}_{\text{fl,fin}} = & \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{A0}} \left[\frac{1}{2} |\dot{f}_A|^2 + \frac{1}{2} (k^2 + \mathcal{M}_A^2(t)) |f_A|^2 - \omega_{A0}^2 - \frac{\mathcal{V}_A}{2} + \frac{\mathcal{V}_A^2}{8\omega_{A0}^2} \right] \\ & + \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{B0}} \left[\frac{1}{2} |\dot{f}_B|^2 + \frac{1}{2} (k^2 + \mathcal{M}_B^2(t)) |f_B|^2 - \omega_{B0}^2 - \frac{\mathcal{V}_B}{2} + \frac{\mathcal{V}_B^2}{8\omega_{B0}^2} \right] \\ & + \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_{\psi0}} \left[i(\omega_{\psi0} - m_{\psi0})(f_\psi \dot{f}_\psi^* - \dot{f}_\psi f_\psi^*) - 2\omega_{\psi0} \mathcal{M}_\psi(t) + 2\omega_{\psi0}^2 + \mathcal{M}_\psi^2(t) \right. \\ & \left. - m_{\psi0}^2 - \frac{1}{4\omega_{\psi0}^2} (\dot{\mathcal{M}}_\psi^2(t) + \mathcal{M}_\psi^4(t) + m_{\psi0}^4 - 2\mathcal{M}_\psi^2(t)m_{\psi0}^2) \right] . \quad (\text{A29}) \end{aligned}$$

The energy has no initial singularity. In addition to the quadratic and logarithmic divergences we find here a quartic one. In dimensional regularization it can be written as

$$I_1(m) = \int \frac{d^3k}{(2\pi)^3} \omega_{j0} = -\frac{m_{j0}^4}{2} I_{-3}(m) - \frac{m_{j0}^4}{2} \ln \frac{m_{j0}^2}{m^2} - \frac{3m_{j0}^4}{64\pi^2} . \quad (\text{A30})$$

In the same way as for the equation of motion we can now evaluate the momentum integrals in \mathcal{E}_{div} , fix the counter terms, and find the finite contributions. This leads to

$$\begin{aligned} \mathcal{E}_{\text{div}} = & \lambda^2 \left[\frac{1}{2} \dot{\phi}^2 - V(\phi) \right] I_{-3}(m) + \Delta F + \Delta \Lambda \\ & + \frac{1}{2} \Delta Z \dot{\phi}^2 + \frac{1}{2} \Delta m \phi^2 + \Delta \lambda \frac{1}{2} m \lambda \phi^3 + \frac{1}{8} \Delta \lambda \lambda^2 \phi^4 , \quad (\text{A31}) \end{aligned}$$

with

$$\begin{aligned} \Delta \Lambda = & \frac{1}{128\pi^2} (m_{A0}^4 + m_{B0}^4 - 2m_{\psi0}^4) \\ & - \frac{m^4}{64\pi^2} \left(\ln \frac{m_{A0}^2}{m^2} + \ln \frac{m_{B0}^2}{m^2} - 2 \ln \frac{m_{\psi0}^2}{m^2} \right) . \quad (\text{A32}) \end{aligned}$$

No divergent “cosmological constant” counter term is needed, as to be expected in a supersymmetric model. After taking into account the counter term Lagrangian (A22) with the previously determined coefficients the energy becomes finite and reads

$$\begin{aligned} \mathcal{E}_{\text{ren}} = & \frac{1}{2} (1 + \Delta Z) \dot{\phi}^2 + \frac{1}{2} (m^2 + \Delta m) \phi^2 + \frac{1}{2} m \lambda (1 + \Delta \lambda) \phi^3 \\ & + \frac{\lambda^2}{8} (1 + \Delta \lambda) \phi^4 + \Delta F \phi + \Delta \Lambda + \mathcal{E}_{\text{fl,fin}} . \quad (\text{A33}) \end{aligned}$$

APPENDIX B: RENORMALIZATION OF THE LARGE- N MODEL

The expressions for the expectation values are analogous to those of the simple Wess-Zumino model, and so is their perturbative expansion. Of course now the fluctuation masses $\mathcal{M}_j(t)$, given by Eqs. (3.10),(3.11) and (3.9), depend on the expectation values of the fluctuations, and so do the potentials $\mathcal{V}_j(t)$, defined in analogy to (A2)-(A4).

In the following we will need the sum and the difference of the bosonic fluctuations. For the sum we obtain

$$\langle A^2 + B^2 \rangle = -2I_{-3}(m^2) (m + \lambda\phi)^2 + \mathcal{F}_{\text{fin}}^+(t) , \quad (\text{B1})$$

with

$$\begin{aligned} \mathcal{F}_{\text{fin}}^+(t) = \frac{1}{16\pi^2} \left\{ \ln \left(\frac{m_{A0}^2}{m^2} \right) \mathcal{M}_A^2(t) + \ln \left(\frac{m_{B0}^2}{m^2} \right) \mathcal{M}_B^2(t) - m_{A0}^2 - m_{B0}^2 \right\} \\ + \langle A^2 \rangle_{\text{fin}} + \langle B^2 \rangle_{\text{fin}} . \end{aligned} \quad (\text{B2})$$

$\langle A^2 \rangle_{\text{fin}}$ and $\langle B^2 \rangle_{\text{fin}}$ are again defined by (A11) and (A12). For the difference we find

$$\langle A^2 - B^2 \rangle = -2I_{-3}(m^2) \left[M\lambda\phi + \frac{1}{2}\kappa\lambda\phi^2 + \frac{1}{2}\lambda^2\langle A^2 - B^2 \rangle \right] + \tilde{\mathcal{F}}_{\text{fin}}^-(t) , \quad (\text{B3})$$

with

$$\begin{aligned} \tilde{\mathcal{F}}_{\text{fin}}^-(t) = \frac{1}{16\pi^2} \left\{ \ln \left(\frac{m_{A0}^2}{m^2} \right) \mathcal{M}_A^2(t) - \ln \left(\frac{m_{B0}^2}{m^2} \right) \mathcal{M}_B^2(t) - m_{A0}^2 + m_{B0}^2 \right\} \\ + \langle A^2 \rangle_{\text{fin}} - \langle B^2 \rangle_{\text{fin}} . \end{aligned} \quad (\text{B4})$$

Obviously the equation for $\langle A^2 - B^2 \rangle$ is implicit, as to be expected in the large- N framework. The logarithms of the ratios m_{j0}^2/m^2 arise from replacing the masses m_{j0} by the common renormalized mass m of all the component fields in the integral $I_{-3}(m^2)$. It is convenient to rewrite these equations in terms of \mathcal{M}_-^2 , as introduced in Eq. (3.9). We get

$$\langle A^2 - B^2 \rangle = -2 \left[I_{-3}(m^2) - \frac{1}{16\pi^2} \ln \left(\frac{m_{A0}m_{B0}}{m^2} \right) \right] \mathcal{M}_-^2 + \mathcal{F}_{\text{fin}}^-(t) , \quad (\text{B5})$$

with

$$\begin{aligned} \mathcal{F}_{\text{fin}}^-(t) = -\frac{1}{16\pi^2} \ln \left(\frac{m_{A0}^2}{m_{B0}^2} \right) \mathcal{M}_\psi^2(t) - \frac{1}{16\pi^2} (m_{A0}^2 - m_{B0}^2) \\ + \langle A^2 \rangle_{\text{fin}} - \langle B^2 \rangle_{\text{fin}} . \end{aligned} \quad (\text{B6})$$

The expectation value of the fermionic fluctuations again decomposes as (A13) and (A14).

We now introduce multiplicative renormalization factors by replacing

$$\lambda \rightarrow Z_\lambda \lambda \quad (\text{B7})$$

$$A \rightarrow Z_A A \quad (\text{B8})$$

$$\phi \rightarrow Z_\phi \phi \quad (\text{B9})$$

and similarly for all other quantities. We first note that there is no divergent term that would require an infinite renormalization of the mass m . This mass occurs in the combination $m + \lambda\phi$ which now gets replaced by $m + Z_\lambda Z_\phi \lambda\phi$. We conclude that also the second term stays unrenormalized, and so

$$Z_\lambda Z_\phi = 1 . \quad (\text{B10})$$

We next analyze the mass $\mathcal{M}_-^2(t)$. We have

$$\mathcal{M}_-^2(t) = Z_M Z_\lambda Z_\phi M \lambda\phi + \frac{1}{2} Z_\kappa Z_\lambda Z_\phi^2 \kappa \lambda\phi^2 + \frac{1}{2} Z_A^2 Z_\lambda^2 \lambda^2 \langle A^2 - B^2 \rangle . \quad (\text{B11})$$

We have assumed $Z_A = Z_B$. Further, from (B5), we have

$$\begin{aligned} \mathcal{M}_-^2 \left[1 + \lambda^2 Z_A^2 Z_\lambda^2 I_{-3}(m^2) \right] &= Z_M Z_\lambda Z_\phi M \lambda\phi + \frac{1}{2} Z_\kappa Z_\lambda Z_\phi^2 \kappa \lambda\phi^2 \\ &+ \lambda^2 Z_A^2 Z_\lambda^2 \frac{1}{16\pi^2} \ln \left(\frac{m_{A0} m_{B0}}{m^2} \right) \mathcal{M}_-^2 + \frac{1}{2} Z_A^2 Z_\lambda^2 \lambda^2 \mathcal{F}_{\text{fin}}^- . \end{aligned} \quad (\text{B12})$$

In order to get analogous relations for the finite quantities as for the unrenormalized ones we require

$$\left[1 + \lambda^2 Z_A^2 Z_\lambda^2 I_{-3}(m^2) \right] = Z_A^2 Z_\lambda^2 = Z_M = Z_\kappa Z_\phi , \quad (\text{B13})$$

having used Eq. (B10). The first relation yields

$$Z_A^2 Z_\lambda^2 = \frac{1}{1 - \lambda^2 I_{-3}(m^2)} . \quad (\text{B14})$$

We then have further

$$Z_M = Z_\kappa Z_\phi = \frac{1}{1 - \lambda^2 I_{-3}(m^2)} . \quad (\text{B15})$$

The renormalized equation for the mass \mathcal{M}_- becomes

$$\mathcal{M}_-^2(t) \left[1 + \frac{\lambda^2}{16\pi^2} \ln \left(\frac{m_{A0} m_{B0}}{m^2} \right) \right] = M \lambda\phi(t) + \frac{1}{2} \kappa \lambda\phi^2(t) + \frac{1}{2} \lambda^2 \mathcal{F}_{\text{fin}}^- , \quad (\text{B16})$$

or

$$\mathcal{M}_-^2(t) = \mathcal{C} \left[M \lambda\phi(t) + \frac{1}{2} \kappa \lambda\phi^2(t) + \frac{1}{2} \lambda^2 \mathcal{F}_{\text{fin}}^- \right] , \quad (\text{B17})$$

with

$$\mathcal{C} = \frac{1}{1 + \frac{\lambda^2}{16\pi^2} \ln \left(\frac{m_{A0} m_{B0}}{m^2} \right)} . \quad (\text{B18})$$

Using Eq. (B10) we also have the renormalized relations

$$\mathcal{M}_A^2(t) = [m + \lambda\phi(t)]^2 + \mathcal{M}_-^2(t) , \quad (\text{B19})$$

$$\mathcal{M}_B^2(t) = [m + \lambda\phi(t)]^2 - \mathcal{M}_-^2(t) , \quad (\text{B20})$$

$$\mathcal{M}_\psi(t) = m + \lambda\phi(t) . \quad (\text{B21})$$

We now turn to the equation of motion for the mean field $\phi(t)$, Eq. (3.4). Introducing the renormalization factors we obtain

$$Z_\phi \ddot{\phi} + \frac{Z_M}{Z_\lambda \lambda} (M + \kappa\phi) \mathcal{M}_-^2 + Z_\lambda^2 Z_\phi Z_\psi^2 \lambda^2 I_{-3}(m^2) \ddot{\phi} \quad (\text{B22})$$

$$+ Z_\lambda Z_A^2 \lambda (m + \lambda\phi) \langle A^2 + B^2 \rangle + Z_\lambda Z_\psi^2 \frac{\lambda}{2} \langle \bar{\Psi} \Psi \rangle = 0 . \quad (\text{B23})$$

We first consider the last two terms. These terms are quadratically divergent. This ‘‘tadpole’’ contribution necessitates a cancellation between bosons and fermions. This works only if we postulate

$$Z_\psi = Z_A = Z_B , \quad (\text{B24})$$

a relation indeed required by supersymmetry. Then

$$Z_\lambda Z_A^2 \lambda (m + \lambda\phi) \langle A^2 + B^2 \rangle + Z_\lambda Z_\psi^2 \frac{\lambda}{2} \langle \bar{\Psi} \Psi \rangle = Z_\lambda Z_A^2 \lambda \left[\mathcal{M}_\psi \langle A^2 + B^2 \rangle + \frac{1}{2} \langle \bar{\Psi} \Psi \rangle \right] . \quad (\text{B25})$$

Inserting the decomposition into divergent parts and fluctuation integrals the expression in brackets yields

$$[\dots] = I_{-3}(m^2) \ddot{\mathcal{M}}_\psi + (m + \lambda\phi) \mathcal{F}_{\text{fin}}^+ + \frac{1}{2} \mathcal{F}_{\text{fin}}^f , \quad (\text{B26})$$

so that the equation of motion for ϕ takes the form

$$Z_\phi \ddot{\phi} + \frac{Z_M}{Z_\lambda \lambda} (M + \kappa\phi) \mathcal{M}_-^2 + Z_\lambda^2 Z_\phi Z_A^2 \lambda^2 I_{-3}(m^2) \ddot{\phi} \quad (\text{B27})$$

$$+ \frac{1}{2} Z_\lambda Z_A^2 \lambda \mathcal{F}_{\text{fin}}^f + Z_\lambda Z_A^2 \lambda (m + \lambda\phi) \mathcal{F}_{\text{fin}}^+ = 0 . \quad (\text{B28})$$

The coefficients of the two terms containing $\ddot{\phi}$ combine into

$$Z_\phi \left[1 + Z_\lambda^2 Z_\psi^2 \lambda^2 I_{-3}(m^2) \right] = Z_\phi \frac{1}{1 - \lambda^2 I_{-3}(m^2)} . \quad (\text{B29})$$

Here we have used Eq. (B14). The factor of the $\ddot{\phi}$ term agrees with that of the second term in Eq. (B27)

$$\frac{Z_M}{Z_\lambda} = Z_M Z_\phi = Z_\phi \frac{1}{1 - \lambda^2 I_{-3}(m^2)} . \quad (\text{B30})$$

Likewise for the fourth term in Eq. (B27) we obtain

$$Z_\lambda Z_A^2 = Z_\phi Z_\lambda^2 Z_A^2 = Z_\phi \frac{1}{1 - \lambda^2 I_{-3}(m^2)} . \quad (\text{B31})$$

The same factor is obtained for the fifth term, so the renormalized equation reads

$$\ddot{\phi} + \frac{1}{\lambda}(M + \kappa\phi)\mathcal{M}_-^2 + \frac{1}{2}\lambda\mathcal{F}_{\text{fin}}^f + \lambda(m + \lambda\phi)\mathcal{F}_{\text{fin}}^+ = 0. \quad (\text{B32})$$

The various factors Z_j are not yet completely determined. This is to be expected, since the equations of motion do not depend on the absolute normalization of the Lagrangian. When including the Z factors and using the various relations derived so far, the energy density is given by

$$\begin{aligned} \mathcal{E} = & \frac{1}{2}Z_\phi^2\dot{\phi}^2 + \frac{1}{2}Z_M^2Z_\phi^2\left(M\phi + \frac{\kappa}{2}\phi^2\right)^2 \\ & + \frac{1}{2}Z_A^2\langle\dot{A}^2 + k^2A^2 + \mathcal{M}_A^2(t)A^2\rangle + \frac{1}{2}Z_A^2\langle\dot{B}^2 + k^2B^2 + \mathcal{M}_B^2(t)B^2\rangle \\ & - Z_\lambda^2Z_A^4\frac{\lambda^2}{8}\left(\langle A^2 - B^2\rangle\right)^2 + Z_A^2\frac{1}{2}\left\langle\bar{\psi}\left(-i\vec{\gamma}\cdot\vec{\nabla} + \mathcal{M}_\psi(t)\right)\psi\right\rangle. \end{aligned} \quad (\text{B33})$$

The bosonic fluctuation energies defined as as

$$\begin{aligned} \mathcal{E}_A = & \frac{1}{2}\langle\dot{A}^2 + k^2A^2 + \mathcal{M}_A^2(t)A^2\rangle \\ = & \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{A0}} \left\{ \frac{1}{2}|\dot{f}_A|^2 + \frac{1}{2}\left[k^2 + \mathcal{M}_A^2(t)\right]|f_A|^2 \right\}, \end{aligned} \quad (\text{B34})$$

and analogously for the field B . Its expansion in terms of divergent integrals reads

$$\begin{aligned} \mathcal{E}_A = & \frac{1}{2}I_1(m_{A0}^2) + \frac{1}{2}I_{-1}(m_{A0}^2)\mathcal{V}_A(t) - \frac{1}{4}I_{-3}(m_{A0}^2)\mathcal{V}_A^2(t) + \mathcal{E}_{A,\text{fin}} \\ = & -\frac{1}{4}I_{-3}(m^2)\mathcal{M}_A^4 + \frac{1}{128\pi^2}m_{A0}^4 - \frac{1}{32\pi^2}m_{A0}^2\mathcal{M}_A^2 - \frac{1}{64\pi^2}\ln\left(\frac{m^2}{m_{A0}^2}\right)\mathcal{M}_A^4 + \mathcal{E}_{A,\text{fin}}. \end{aligned} \quad (\text{B35})$$

Here we have used Eqs. (A9), (A10), and (A30). The finite part is defined by subtracting the quantity

$$\mathcal{E}_{A,\text{div}} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{A0}} \left\{ \omega_{A0}^2 + \frac{1}{2}\mathcal{V}_A - \frac{1}{8\omega_{A0}^2}\mathcal{V}_A^2 \right\}, \quad (\text{B36})$$

from the expression (B34).

The fermionic fluctuation energy is defined as

$$\begin{aligned} \mathcal{E}_\psi = & \frac{1}{2}\left\langle\bar{\psi}\left(-i\vec{\gamma}\cdot\vec{\nabla} + \mathcal{M}_\psi(t)\right)\psi\right\rangle \\ = & \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{\psi0}} \left\{ i\left[\omega_{\psi0} - m_{\psi0}\right]\left(f(k,t)\dot{f}^*(k,t) - \dot{f}(k,t)f^*(k,t)\right) - 2\omega_{\psi0}\mathcal{M}_\psi(t) \right\}. \end{aligned} \quad (\text{B37})$$

It is expanded as

$$\begin{aligned} \mathcal{E}_\psi = & \frac{1}{2}\left[\dot{\mathcal{M}}_\psi^2 + \mathcal{M}_\psi^4\right]\left[I_{-3}(m^2) + \frac{1}{16\pi^2}\ln\left(\frac{m^2}{m_{\psi0}^2}\right)\right] \\ & - \frac{m_{\psi0}^4}{64\pi^2} + \frac{m_{\psi0}^2}{16\pi^2}\mathcal{M}_\psi^2 + \mathcal{E}_{\psi,\text{fin}}, \end{aligned} \quad (\text{B38})$$

where $\mathcal{E}_{\psi,\text{fin}}$ is defined by subtracting

$$\mathcal{E}_{\psi,\text{div}} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{\psi 0}} \left\{ -2\omega_{\psi 0}^2 - (\mathcal{M}_{\psi}^2 - m_{\psi 0}^2) + \frac{\dot{\mathcal{M}}_{\psi}^2}{4\omega_{\psi 0}^2} + \frac{(\mathcal{M}_{\psi}^2 - m_{\psi 0}^2)^2}{4\omega_{\psi 0}^2} \right\}, \quad (\text{B39})$$

from the integral of Eq. (B37).

With these preparations we are ready to discuss the divergences of the energy density.

The term proportional to $\dot{\mathcal{M}}_{\psi}^2 = \lambda^2 \dot{\phi}^2$ adds to the kinetic term $Z_{\phi}^2 \dot{\phi}^2$. The total factor (up to a finite term, see below) is $Z_{\phi}^2 + Z_A^2 \lambda^2 I_{-3}$. If we compare to the relations for the Z_j found in the previous section we find that the prefactor of $\dot{\phi}^2$ becomes finite if we choose $Z_A = 1$ and as a consequence $Z_{\phi}^2 = 1 - \lambda^2 I_{-3}$.

The second and the fifth term in the general expression for the energy, Eq. (B33) can be rewritten, using Eq. (B11) as

$$\begin{aligned} & \frac{1}{2} Z_M^2 Z_{\phi}^2 \left(M\phi + \frac{\kappa}{2} \phi^2 \right)^2 - Z_{\lambda}^2 Z_A^4 \frac{\lambda^2}{8} (\langle A^2 - B^2 \rangle)^2 \\ &= \frac{1}{2} \left[Z_M Z_{\phi} \left(M\phi + \frac{\kappa}{2} \phi^2 \right) - \frac{\lambda}{2} Z_{\lambda} Z_A^2 \langle A^2 - B^2 \rangle \right] \frac{1}{Z_{\lambda} \lambda} \mathcal{M}_{-}^2 \\ &= Z_M Z_{\phi} \left(M\phi + \frac{\kappa}{2} \phi^2 \right) \frac{1}{Z_{\lambda} \lambda} \mathcal{M}_{-}^2 - \frac{1}{2 Z_{\lambda}^2 \lambda^2} \mathcal{M}_{-}^4. \end{aligned} \quad (\text{B40})$$

We note that $Z_M Z_{\phi} / Z_{\lambda} = 1$ for the choice (B43), so that the first term on the right hand side is finite. The second term $-\mathcal{M}_{-}^4 / 2 Z_{\lambda}^2 \lambda^2$ has a factor $Z_{\lambda}^{-2} = 1 - \lambda^2 I_{-3}$. This combines with the infinite parts of $\mathcal{E}_A, \mathcal{E}_B$ and \mathcal{E}_{ψ} as

$$\begin{aligned} & -\frac{1}{2} \mathcal{M}_{-}^4 \left[\frac{1}{\lambda^2} - I_{-3}(m^2) \right] - \frac{1}{4} (\mathcal{M}_{\psi}^2 + \mathcal{M}_{-}^2)^2 I_{-3}(m^2) \\ & -\frac{1}{4} (\mathcal{M}_{\psi}^2 - \mathcal{M}_{-}^2)^2 I_{-3}(m^2) + \frac{1}{2} \mathcal{M}_{\psi}^4 I_{-3}(m^2) = -\frac{\mathcal{M}_{-}^4}{2\lambda^2}. \end{aligned} \quad (\text{B41})$$

So all infinite terms have cancelled, and the choice of renormalization constants

$$Z_A = Z_B = Z_{\psi} = 1, \quad (\text{B42})$$

$$Z_{\lambda}^2 = Z_{\phi}^{-2} = Z_M = \frac{1}{1 - \lambda^2 I_{-3}(m^2)}, \quad (\text{B43})$$

$$Z_{\kappa} = \frac{1}{[1 - \lambda^2 I_{-3}(m^2)]^{3/2}}, \quad (\text{B44})$$

based on the analysis of the equations of motion and of the energy momentum tensor. It remains to collect the finite terms. We find

$$\begin{aligned} \mathcal{E} &= \frac{1}{2} \dot{\phi}^2 \left[1 - \frac{\lambda^2}{32\pi^2} \ln \left(\frac{m_{\psi 0}^2}{m^2} \right) \right] + \left(M\phi + \frac{\kappa}{2} \phi^2 \right) \frac{\mathcal{M}_{-}^2}{\lambda} - \frac{1}{2\lambda^2} \mathcal{M}_{-}^4 \\ &+ \tilde{\mathcal{E}}_{A,\text{fin}} + \tilde{\mathcal{E}}_{B,\text{fin}} + \tilde{\mathcal{E}}_{\psi,\text{fin}}, \end{aligned} \quad (\text{B45})$$

with

$$\tilde{\mathcal{E}}_{A,\text{fin}} = \mathcal{E}_{A,\text{fin}} + \frac{1}{128\pi^2} \left[m_{A0}^4 - 4m_{A0}^2 \mathcal{M}_A^2 + 2 \ln \left(\frac{m_{A0}^2}{m^2} \right) \mathcal{M}_A^4 \right] \quad (\text{B46})$$

$$\tilde{\mathcal{E}}_{B,\text{fin}} = \mathcal{E}_{B,\text{fin}} + \frac{1}{128\pi^2} \left[m_{B0}^4 - 4m_{B0}^2 \mathcal{M}_B^2 + 2 \ln \left(\frac{m_{B0}^2}{m^2} \right) \mathcal{M}_B^4 \right] \quad (\text{B47})$$

$$\tilde{\mathcal{E}}_{\psi,\text{fin}} = \mathcal{E}_{\psi,\text{fin}} - \frac{1}{64\pi^2} \left[m_{\psi0}^4 - 4m_{\psi0}^2 \mathcal{M}_\psi^2 + 2 \ln \left(\frac{m_{\psi0}^2}{m^2} \right) \mathcal{M}_\psi^4 \right] . \quad (\text{B48})$$

We note again that no ‘cosmological constant’ counter term has to be introduced.

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